THERMOVISCOELASTICITY EQUATIONS OF A COSSERAT MEDIUM WITH THERMAL MEMORY

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UDC 539.376

A system of equations of nonsymmetric thermoviscoelasticity is obtained for the displacement and rotation vectors with thermal memory taken into account.

Interest in the thermomechanics of a continuous medium, which is distinguished from a classical medium by the more complex properties, is observed at this time. We speak about models that the account of the microstructure of real materials. The simplest of such models is the Cosserat continuum. A modern version of the theory of elasticity of a Cosserat medium is obtained in [1, 2]. Later, a related theory of thermoelasticity was developed in [3], and equations for generalized thermoelasticity of a Cosserat medium are obtained in [4]. A further development of the theory of thermodeformation of such a medium requires taking account of the presence of memory.

It is known that the equations of motion of a Cosserat medium have the form

$$\nabla\cdot\mathbf{T}+\rho\mathbf{X}=\rho\ddot{\mathbf{u}};\ \nabla\cdot\mathbf{M}-2\mathbf{a}^{T}+\rho\mathbf{Y}=\mathbf{I}\cdot\ddot{\boldsymbol{\omega}}, \tag{1}$$

where $\mathbf{a}^{T}$ is the vector accompanying the tensor $\mathbf{T}$. To describe the deformation we introduce the nonsymmetric deformation tensor $\boldsymbol{\gamma}$ and the bending–torsion tensor $\boldsymbol{\varkappa}$, connected to the displacement and rotation vectors by the following relationships

$$\boldsymbol{\gamma}=\nabla\mathbf{u}+\boldsymbol{\omega}\times\mathbf{E};\ \boldsymbol{\varkappa}=\nabla\boldsymbol{\omega}. \tag{2}$$

Consideration of the thermal processes requires the introduction of the internal energy of a material volume h and the internal entropy of a material volume s which are additive functions of the mass expressed in terms of its density:

$$h=\int\limits_{V}\rho H dV;\ s=\int\limits_{V}\rho S dV. \tag{3}$$

We introduce the laws of thermodynamics in the form just as is done in [5, 6]. The first has the form

$$\dot{K}+\dot{h}=A+Q, \tag{4}$$

where

$$A=\int\limits_{V}(\mathbf{X}\cdot\dot{\mathbf{u}}+\mathbf{Y}\cdot\dot{\boldsymbol{\omega}})dV+\int\limits_{O}(\mathbf{n}\cdot\mathbf{T}\cdot\dot{\mathbf{u}}+\mathbf{n}\cdot\mathbf{M}\cdot\dot{\boldsymbol{\omega}})dO;$$

$$Q=\int\limits_{V}w dV-\int\limits_{O}\mathbf{n}\cdot\mathbf{q} dO;\ K=\frac{1}{2}\int\limits_{V}(\rho\dot{\mathbf{u}}\cdot\dot{\mathbf{u}}+\dot{\boldsymbol{\omega}}\cdot\mathbf{I}\cdot\dot{\boldsymbol{\omega}})dV. \tag{5}$$

The second law of thermodynamics is written thus:

$$\dot{s}\geqslant\int\limits_{V}\frac{1}{\Theta}\rho w dV-\int\limits_{O}\frac{1}{\Theta}\mathbf{n}\cdot\mathbf{q} dO. \tag{6}$$

If the free energy density

$$F=H-\Theta S, \tag{7}$$

is introduced into the consideration, then after simple manipulations the laws of thermodynamics can be reduced to the form

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 4, pp. 665-670, April, 1982. Original article submitted January 15, 1981.

0022-0841/82/4204-0466$07.50 © 1982 Plenum Publishing Corporation

$$\mathbf{T}\cdot\cdot\dot{\boldsymbol{\gamma}} + \mathbf{M}\cdot\cdot\dot{\boldsymbol{\varkappa}} - \rho\dot{\Theta}S - \rho\dot{F} - \rho\Theta\dot{S} + \rho w - \nabla\cdot\mathbf{q} = 0;$$
$$\mathbf{T}\cdot\cdot\dot{\boldsymbol{\gamma}} + \mathbf{M}\cdot\cdot\dot{\boldsymbol{\varkappa}} - \rho\dot{\Theta}S - \rho\dot{F} - \frac{1}{\Theta}\mathbf{q}\cdot\nabla\Theta \geqslant 0. \tag{8}$$

It should be noted that the laws (1) and (8) are sufficiently general in nature. The examination of a specific medium requires the assignment of the expression for the free energy. The assignment of a specific form of F is substantially the definition of the medium.

It is assumed in the investigation of viscoelastic media that the free energy is determined by the history of the body deformation. The introduction of a dependence of the free energy on the history of the deformation can be realized by different methods.

In particular, it can be done thus. If it is said that the function f(t) is defined by the history of the change in the function g(t) in the interval (t_0, t), then in a first approximation by separating the interval into N parts it can be said that f is defined by the N arguments g_k:

$$f = f(g_1, \ldots g_k, \ldots g_N);\ g_k = \dot{g}(t_0 + k\Delta t)\,\Delta t;\ \Delta t = \frac{t - t_0}{N}. \tag{9}$$

Expanding f in a power series in g_k and limiting ourselves to an examination of terms not above the second order, we obtain

$$f \approx a + \sum_{k=1}^{N} b_{N-k} g_k + \sum_{k=1}^{N}\sum_{m=1}^{N} c_{N-k,N-m} g_k g_m. \tag{10}$$

Continuing the interval partition process and passing to the limit, we obtain the following expression

$$f = a_0 + \int_{t_0}^{t} b_0(t-\tau)\,\dot{g}(\tau)\,d\tau + \int_{t_0}^{t}\int_{t_0}^{t} c_0(t-\tau,\ t-\eta)\,\dot{g}(\tau)\,\dot{g}(\eta)\,d\tau d\eta. \tag{11}$$

It is here understood that

$$c_0(x,\ y) = c_0(y,\ x). \tag{12}$$

It should be noted that when we speak of the deformation of a medium there is not always a foundation for limiting the history to some finite lower limit. Consequently, for generality, the lower limit is taken at $-\infty$ in many cases. However, as a rule it is necessary to deal with situations when the body is not subjected to the action of external forces and heat sources up to a certain specific time in the solution of specific problems, and there are no strains and stresses. In this case it is appropriate to take this time as the origin.

And finally, still another remark. In considering viscoelastic media it is customary to extract the global and deviator parts in the strain and stress tensors. This is related to the fact that the governing equation, connecting the global parts of the stress and strain tensors for many materials, has the same form as for an elastic medium. Without making such an assumption about the properties of a material, we nevertheless separate the tensors under consideration into global and deviator parts

$$\mathbf{T} = L\mathbf{E} + \boldsymbol{\Gamma};\ \mathbf{M} = J\mathbf{E} + \boldsymbol{\Pi};\ \boldsymbol{\gamma} = G\mathbf{E} + \boldsymbol{\Phi};\ \boldsymbol{\varkappa} = W\mathbf{E} + \boldsymbol{\Omega}. \tag{13}$$

This is convenient for the subsequent investigation of particular cases, and also permits conservation of the traditional form of the thermoviscoelasticity equations.

Limiting ourselves to the consideration of an isotropic polar-symmetric medium and taking account of the remarks made, we take the following expression for the free energy of a Cosserat thermoviscoelastic linear medium

$$F = \int_0^t \{-C_1(\chi)\,\dot{\Theta}(\tau) + C_2(\chi)\,\dot{G}(\tau)\}\,d\tau + \int_0^t\int_0^t \{-B_1(\chi,\ \psi)\,\dot{\Theta}(\tau)\,\dot{\Theta}(\eta) +$$
$$+ B_2(\chi,\ \psi)\,\dot{G}(\tau)\,\dot{G}(\eta) - B_3(\chi,\ \psi)\,\dot{\Theta}(\tau)\,\dot{G}(\eta) + B_4(\chi,\ \psi)\,\dot{W}(\tau)\,\dot{W}(\eta) + \tag{14}$$
$$+ B_5(\chi,\ \psi)\,\dot{\boldsymbol{\Phi}}^+(\tau)\cdot\cdot\dot{\boldsymbol{\Phi}}^+(\eta) + B_6(\chi,\ \psi)\,\dot{\boldsymbol{\Phi}}^-(\tau)\cdot\cdot\dot{\boldsymbol{\Phi}}^-(\eta) + B_7(\chi,\ \psi)\,\dot{\boldsymbol{\Omega}}^+(\tau)\cdot\cdot\dot{\boldsymbol{\Omega}}^+(\eta) + B_8(\chi,\ \psi)\,\dot{\boldsymbol{\Omega}}^-(\tau)\cdot\cdot\dot{\boldsymbol{\Omega}}^-(\eta)\}\,d\eta d\tau,$$

where

$$\chi = t - \tau;\ \psi = t - \eta. \tag{15}$$

Substituting the expression for the free energy in the dissipation inequality (8), we obtain

$$
\begin{gathered}
-\dot{\Theta}\left\{\rho S-\rho C_1(0)-2\rho\int_0^t\{B_1(\chi,\ 0)\,\dot{\Theta}(\tau)+B_3(\chi,\ 0)\,\dot{G}(\tau)\}\,d\tau\right\}+\\
+\dot{W}\left\{3J-2\rho\int_0^t B_4(\chi,\ 0)\,\dot{W}(\tau)\,d\tau\right\}+\dot{\mathbf{\Omega}}\cdot\cdot\left\{\mathbf{\Pi}-2\rho\int_0^t\{B_7(\chi,\ 0)\,\dot{\mathbf{\Omega}}^{+}(\tau)+\right.\\
\left.+B_8(\chi,\ 0)\,\dot{\mathbf{\Omega}}^{-}(\tau)\}\,d\tau\right\}+\dot{G}\left\{3L-\rho C_2(0)+\rho\int_0^t(B_3(\chi,\ 0)\,\dot{\Theta}(\tau)-2B_2(\chi,\ 0)\,\dot{G}(\tau)\}\,d\tau\right\}+\dot{\mathbf{\Phi}}\cdot\cdot\left\{\mathbf{\Gamma}-2\rho\int_0^t\{B_5(\chi,\ 0)\,\dot{\mathbf{\Phi}}^{+}(\tau)+\right.\\
\left.+B_6(\chi,\ 0)\,\dot{\mathbf{\Phi}}^{-}(\tau)\}\,d\tau\right\}-\rho D-\frac{1}{\Theta}\mathbf{q}\cdot\nabla\Theta\geqslant 0,
\end{gathered}
\tag{16}
$$

where we have introduced the notation

$$
\begin{gathered}
D=\int_0^t\frac{\partial}{\partial t}\{-C_1(\chi)\,\dot{\Theta}(\tau)+C_2(\chi)\,\dot{G}(\tau)\}\,d\tau+\int_0^t\int_0^t\frac{\partial}{\partial t}\{-B_1(\chi,\ \psi)\,\dot{\Theta}(\tau)\,\dot{\Theta}(\eta)+\\
+B_2(\chi,\ \psi)\,\dot{G}(\tau)\,\dot{G}(\eta)-B_3(\chi,\ \psi)\,\dot{\Theta}(\tau)\,\dot{G}(\eta)+B_4(\chi,\ \psi)\,\dot{W}(\tau)\,\dot{W}(\eta)+B_5(\chi,\ \psi)\,\dot{\mathbf{\Phi}}^{+}(\tau)\cdot\cdot\dot{\mathbf{\Phi}}^{+}(\eta)+B_6(\chi,\ \psi)\,\dot{\mathbf{\Phi}}^{-}(\tau)\cdot\cdot\dot{\mathbf{\Phi}}^{-}(\eta)\\
+B_7(\chi,\ \psi)\,\dot{\mathbf{\Omega}}^{+}(\tau)\cdot\cdot\dot{\mathbf{\Omega}}^{+}(\eta)+B_8(\chi,\ \psi)\,\dot{\mathbf{\Omega}}^{-}(\tau)\cdot\cdot\dot{\mathbf{\Omega}}^{-}(\eta)\}\,d\eta\,d\tau.
\end{gathered}
\tag{17}
$$

It is easy to see that a corollary of (16) will be the system of governing equations

$$
\begin{gathered}
S=\int_0^t\{l_3(\chi)\,\dot{G}(\tau)+2l_1(\chi)\,\dot{\Theta}(\tau)\}\,d\tau+C_1(0);\\
J=\frac{2}{3}\rho\int_0^t l_4(\chi)\,\dot{W}(\tau)\,d\tau;\\
\mathbf{\Pi}=2\rho\int_0^t\{l_7(\chi)\,\dot{\mathbf{\Omega}}^{+}(\tau)+l_8(\chi)\,\dot{\mathbf{\Omega}}^{-}(\tau)\}\,d\tau;\\
L=\frac{1}{3}\rho\int_0^t\{-l_3(\chi)\,\dot{\Theta}(\tau)+2l_2(\chi)\,\dot{G}(\tau)\}\,d\tau+\rho C_2(0);\\
\mathbf{\Gamma}=2\rho\int_0^t\{l_5(\chi)\,\dot{\mathbf{\Phi}}^{+}(\tau)+l_6(\chi)\,\dot{\mathbf{\Phi}}^{-}(\tau)\}\,d\tau
\end{gathered}
\tag{18}
$$

and the inequality

$$
-\rho D-\frac{1}{\Theta}\mathbf{q}\cdot\nabla\Theta\geqslant 0,
\tag{19}
$$

where

$$
l_k(\chi)=B_k(\chi,\ 0).
\tag{20}
$$

If the internal entropy is measured from the level corresponding to the undeformed state S(0), then by assuming

$$
T|_{t=0}=0,
\tag{21}
$$

we obtain

$$
C_1(0)=0;\ C_2(0)=0.
\tag{22}
$$

Now, if it is assumed that the relaxation functions are monotonic, nonnegative, and not growing, in agreement with the physical representations, then it follows from (22) that

$$
C_1(x)=0;\ C_2(x)=0.
\tag{23}
$$

Let us now use the definition of the strain and bending–torsion tensors (2), the governing equations (14), and let us eliminate the stress tensor from the equations of motion (1). We consequently obtain a system of equations in the displacement and rotation vectors

$$
\begin{gathered}
\rho\int_0^t\left\{[l_5(\chi)+l_6(\chi)]\,\nabla^2\dot{\mathbf{u}}(\tau)+\frac{1}{9}[2l_2(\chi)+3l_5(\chi)-9l_6(\chi)]\,\nabla\nabla\cdot\dot{\mathbf{u}}(\tau)-\frac{1}{3}l_3(\chi)\,\nabla\dot{\Theta}(\tau)+2l_6(\chi)\,\nabla\times\dot{\boldsymbol{\omega}}(\tau)\right\}d\tau+\rho\mathbf{X}=\rho\ddot{\mathbf{u}};\\
\rho\int_0^t\left\{[l_7(\chi)+l_8(\chi)]\,\nabla^2\dot{\boldsymbol{\omega}}(\tau)+\frac{1}{9}[2l_4(\chi)+3l_7(\chi)-9l_8(\chi)]\,\nabla\nabla\cdot\dot{\boldsymbol{\omega}}(\tau)+2l_6(\chi)\,\nabla\times\dot{\mathbf{u}}(\tau)-4l_6(\chi)\,\dot{\boldsymbol{\omega}}(\tau)\right\}d\tau+\rho\mathbf{Y}=\mathbf{I}\cdot\ddot{\boldsymbol{\omega}}.
\end{gathered}
\tag{24}
$$

The system of equations obtained should be supplemented by the heat conduction equation. We take the heat conduction law in a form taking account of the presence of a thermal memory

$$\mathbf{q} = -\int_0^t b(\chi)\,\nabla\dot{\Theta}(\tau)\,d\tau. \tag{25}$$

If the expression for F is substituted into the equation for the first law of thermodynamics (8), then taking account of the governing equations (18), we obtain

$$\rho\Theta\dot{S} = \rho w - \nabla\cdot\mathbf{q} - \rho D. \tag{26}$$

Let us introduce the relative temperature deviation

$$\vartheta = (\Theta - \Theta_0)\,\Theta_0^{-1}. \tag{27}$$

The equations (24) are linear. Invariance of the equations of motion with respect to the measured and the actual configurations are used in their derivation, and terms not higher than the quadratic are used in the expression for the free energy. Therefore, (26) must also be linearized. Eliminating the internal entropy density from (26) by using the governing equation, and the thermal flux vector by using the heat conduction law, by conserving only linear terms we obtain the heat conduction equation

$$\int_0^t \left\{ b(\chi)\,\nabla^2\dot{\vartheta}(\tau) - \frac{\partial}{\partial t}\left\{2l_1(\chi)\,\dot{\vartheta}(\tau) + \frac{1}{3}\Theta_0^{-1}\,l_3(\chi)\,\nabla\cdot\dot{\mathbf{u}}(\tau)\right\}\rho\Theta_0\right\} d\tau - \rho\Theta_0\left(2l_1(0)\,\dot{\vartheta}(t) + \frac{1}{3}\Theta_0^{-1}l_3(0)\,\nabla\cdot\dot{\mathbf{u}}(t)\right) = -\rho\Theta_0^{-1}w. \tag{28}$$

The system (24) and (28) is the complete system of thermoviscoelasticity equations of a Cosserat medium with thermal memory taken into account.

Let us examine the following problem: Let forces X, moments Y, heat liberation sources w act on a body bounded by the surface O, and let the following boundary and initial conditions hold:

$$\begin{gathered} \mathbf{u}\big|_O = \mathbf{u}_0(t); \quad \boldsymbol{\omega}\big|_O = \boldsymbol{\omega}_0(t); \quad \vartheta\big|_O = \vartheta_0(t); \\ \mathbf{u}\big|_{t=0} = 0; \quad \dot{\mathbf{u}}\big|_{t=0} = 0; \quad \boldsymbol{\omega}\big|_{t=0} = 0; \quad \dot{\boldsymbol{\omega}}\big|_{t=0} = 0; \quad \vartheta\big|_{t=0} = 0. \end{gathered} \tag{29}$$

Using the Laplace transform, we reduce the system (24) and (28) to the form

$$\begin{gathered} \rho p\,[\bar{l}_5 + \bar{l}_6]\,\nabla^2\bar{\mathbf{u}} + \frac{\rho p}{9}\,[2\bar{l}_2 + 3\bar{l}_5 - 9\bar{l}_6]\,\nabla\nabla\cdot\bar{\mathbf{u}} - \frac{1}{3}\rho p\Theta_0\bar{l}_3\nabla\bar{\vartheta} + 2\rho p\bar{l}_6\nabla\mathrm{x}\bar{\boldsymbol{\omega}} + \rho\bar{\mathbf{X}} = \rho p^2\bar{\mathbf{u}}; \\ \rho p\,[\bar{l}_7 + \bar{l}_8]\,\nabla^2\bar{\boldsymbol{\omega}} + \frac{\rho p}{9}\,[2\bar{l}_4 + 3\bar{l}_7 - 9\bar{l}_8]\,\nabla\nabla\cdot\bar{\boldsymbol{\omega}} + 2\rho p\bar{l}_6\nabla\mathrm{x}\bar{\mathbf{u}} - 4\rho p\bar{l}_6\,\bar{\boldsymbol{\omega}} + p\bar{\mathbf{Y}} = p^2\mathbf{I}\cdot\bar{\boldsymbol{\omega}}; \\ \rho^{-1}p\bar{b}\nabla^2\bar{\vartheta} - 2p^2\Theta_0\bar{l}_1\,\bar{\vartheta} - \frac{1}{3}\,p^2\bar{l}_3\nabla\cdot\bar{\mathbf{u}} = -\Theta_0^{-1}\bar{w}. \end{gathered} \tag{30}$$

Comparing these equations with the thermoelasticity equations for a Cosserat medium [7], in a Laplace transformation:

$$\begin{gathered} (\mu + \alpha)\,\nabla^2\bar{\mathbf{u}} + (\lambda + \mu - \alpha)\,\nabla\nabla\cdot\bar{\mathbf{u}} + 2\alpha\nabla\mathrm{x}\bar{\boldsymbol{\omega}} - \nu\Theta_0\overline{\nabla\bar{\vartheta}} + \rho\bar{\mathbf{X}} = \rho p^2\bar{\mathbf{u}}; \\ (\gamma + \varepsilon)\,\nabla^2\bar{\boldsymbol{\omega}} + (\beta + \gamma - \varepsilon)\,\nabla\nabla\cdot\bar{\boldsymbol{\omega}} + 2\alpha\nabla\mathrm{x}\bar{\mathbf{u}} - 4\alpha\bar{\boldsymbol{\omega}} + \rho\bar{\mathbf{Y}} = p^2\mathbf{I}\cdot\bar{\boldsymbol{\omega}}; \\ k\nabla^2\bar{\vartheta} - pm\Theta_0\bar{\vartheta} - p\nu\nabla\cdot\bar{\mathbf{u}} = -\Theta_0^{-1}\rho w, \end{gathered} \tag{31}$$

we see that the solution of the system (30) that satisfies (29) will agree with the corresponding solution of the system (31) if the following change of parameters is executed in the latter

$$\begin{gathered} \mu \to \rho p\bar{l}_5; \quad \alpha \to \rho p\bar{l}_6; \quad \lambda \to \frac{2}{9}\rho p\,(\bar{l}_2 - 3\bar{l}_5); \\ \nu \to \frac{1}{3}\rho p\bar{l}_3; \quad \gamma \to \rho p\bar{l}_7; \quad \varepsilon \to \rho p\bar{l}_8; \\ \beta \to \frac{2}{9}\rho p\,(\bar{l}_4 - 3\bar{l}_7); \quad m \to 2\rho p\bar{l}_1; \quad k \to p\bar{b}. \end{gathered} \tag{32}$$

Therefore, a procedure for constructing the solution of the thermoviscoelasticity boundary value problem for a Cosserat medium occurs. For this the known solution of the thermoelasticity problem for such a medium must be used. It must be subjected to the Laplace transform, the change of parameters (32) must be performed, and then the original must be found. The procedure elucidated is an extension of the widely known [8] elastic-viscoelastic analogy.

NOTATION

T, stress force tensor; **M**, moment stress tensor; **u**, displacement vector; **ω**, rotation vector; **X**, external mass force vector; Y, external mass moment vector; ρ, density; I, tensor characterizing the inertial properties of the medium during rotation; H, internal energy density; F, free energy density; S, internal entropy density; K, kinetic energy; A, power of the external mechanical forces; Q, power of the external thermal sources; q, thermal flux vector; w, heat liberation density; Θ, absolute temperature; Θ_0, initial temperature; E, unit tensor; a^B, vector accompanying the tensor **B**; $\mathbf{B}^+$, symmetric part of the tensor **B**; $\mathbf{B}^-$, antisymmetric part of the tensor B; $\bar{f}$, Laplace transform in f.

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EQUIVALENCE OF CERTAIN TYPES OF RHEOLOGICAL EQUATIONS OF STATE FOR POLYMER MEDIA

PART 1. GENERAL ANALYSIS

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UDC 532.135

The conditions are established under which rheological relaxation equations and rheological integral equations will be equivalent.

According to the classification proposed by C. Truesdell and W. Noll [1, 2], the rheological equations of state for aftereffect media fall into three groups: differential equations, relaxation (or strain rate) equations, and integral equations. Equations of the differential type are applicable only to flow with a small Deborah number, i.e., to fluids with a relaxation time much shorter than the time scale of flow. In other cases one must use either relaxation equations or integral equations of state. Many relaxation equations and integral equations of state have already been proposed [4-7]. As a rule, they are partly based on microscopic models of polymer fluids and on certain assumptions regarding the motion of the medium. They also include several parameters which must be evaluated empirically for any specific material. The rheological equations more or less agree with experiments. According to the bibliography on this subject, however, none of them adequately describes the rheological characteristics of various fluids in complex transient strain states. This makes it necessary to try various models for a given material and then, after comparison with the experiment, select the most applicable ones. Such a diversity of rheological equations for fluids with memory impedes the programming of numerical solution of hydrodynamic and thermal problems for rheologically complex fluids. In the case of relaxation equations of state one formulates the problems of hydrodynamics and heat transfer in the form of partial differential equations, which can be solved by conventional methods of finite differences. For integral

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Belorussian Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 4, pp. 670-677, April, 1982. Original article submitted October 10, 1981.

0022-0841/82/4204-0470$07.50 © 1982 Plenum Publishing Corporation